# PET ENGINEERING COLLEGE

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## DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

## UNIT - II

**CLASS** : **S5 ECE**

**SUBJECT CODE** : **EC3551**

**SUBJECT NAME** : **TRANSMISSION LINES AND RF SYSTEMS**

**REGULATION** : **2021**

23.08.23

# Unit - 2

## High frequency transmission line.

Introduction :- Transmission line at RF

When a line is either open wire or co-axial used at radio frequency (RF) assumption are made.

(i) The current (I) is considered to flow on surface of conductor in a skin of very small depth. At very high frequency L will be zero.

(ii) $\omega L >> R$ zero dissipation line

(iii) $G = 0$ $G \rightarrow$ conductance.

## * Open wire :

1. Inductance : $L = \frac{\mu_0}{2\pi} \ln\left(\frac{d}{a}\right)$ H/m

where

d ⇒ radius of wire

a ⇒ distance between 2 wire

$\mu_0 = 4\pi \times 10^{-7}$ H/m

ii) Capacitance.

$$C = \frac{\pi \varepsilon_0}{\ln\left(\frac{d}{a}\right)} \ F/m$$

where, $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

iii) Resistance:

$$R_{dc} = \frac{K}{\pi a^2}$$

$k = \frac{1}{\sigma}$ (conductivity)

$$R_{ac} = \frac{K}{2\pi a \delta}$$

$\delta$ = skin depth

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

* Co-axial line:

1. Inductance:

$$L = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) H/m$$

where,

$a \Rightarrow$ outer radius of inner conductor

$b \Rightarrow$ inner radius of outer conductor

$$\mu_0 = 4\pi \times 10^{-7}\ H/m$$

ii) Capacitance:

$$C = \frac{2\pi\varepsilon}{\ln\left(b/a\right)}\ F/m$$

where, $\varepsilon_0 = 8.854 \times 10^{-12}\ F/m$ $\quad \varepsilon = \varepsilon_0 \varepsilon_r$

iii) Resistance:

$$R_{dc} = \frac{1}{\pi\sigma}\left[\frac{1}{a^2} + \frac{1}{c^2 - b^2}\right]$$

$$R_{ac} = \frac{1}{2\pi a\delta}\left[\frac{1}{a} + \frac{1}{b}\right]$$

$\delta$ = Skin effect

$$\delta = \frac{1}{\sqrt{\pi f \mu\sigma}}$$

where, $c \Rightarrow$ outer radius of outer conductor.

3 Line constant for zero dissipation less line (or) zero dissipation line:

* characteristic impedance $Z_0 = \sqrt{\frac{Z}{Y}}$

$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\begin{bmatrix} \omega L >> R \\ G >> R \end{bmatrix}$$

$$= \sqrt{\frac{j\omega L}{j\omega C}}$$

$$Z_0 = \sqrt{L/C}$$

*Propagation constant

$$\gamma = \sqrt{ZY}$$

R & G neglect

$$= \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \sqrt{j^2 \omega^2 LC} = j\omega\sqrt{LC}$$

$$\alpha + j\beta = j\omega\sqrt{LC}$$

$$\alpha = 0$$

$$\beta = \omega\sqrt{LC}$$

*Velocity of propagation $V_p = \omega/\beta$

$$= \frac{\omega}{\omega\sqrt{LC}}$$

$$V_p = 1/\sqrt{LC}$$

*Wavelength $\lambda = \dfrac{2\pi}{\beta}$

$$\lambda = \frac{2\pi}{\omega\sqrt{LC}}$$

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4. Voltage and Current on the dissipation less line.

$$E = \frac{E_R}{2Z_R}(Z_0 + Z_R)\left(e^{\gamma s} + k e^{-\gamma s}\right),$$

$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$I = \frac{I_R}{2Z_0}(Z_0 + Z_R)\left(e^{\gamma s} - k e^{-\gamma s}\right)$$

By rearranging,

$$E = E_R \cosh \gamma s + I_R Z_0 \sinh \gamma s$$

$$I = I_R \cosh \gamma s + \frac{E_R}{Z_0} \sinh \gamma s$$

For dissipation less line,

$$\alpha = 0, \quad \gamma = j\beta \; ; \; Z_0 = R_0$$

$$E = E_R \cosh j\beta s + I_R R_0 \sinh j\beta s$$

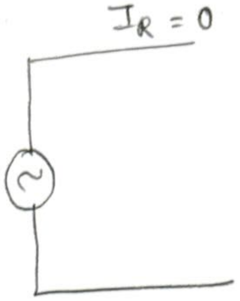
$$I = I_R \cosh j\beta s + \frac{E_R}{R_0} \sinh j\beta s$$

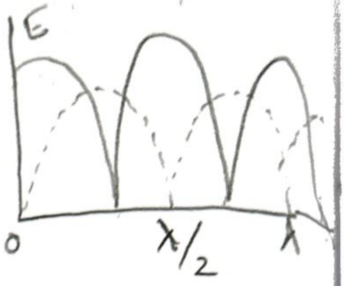
$$\cosh j\beta s = \cos \beta s$$

$$\sinh j\beta s = j \sin \beta s$$

$$\left.\begin{aligned} E &= E_R \cos\beta s + I_R R_0 \, j\sin\beta s \\ I &= I_R \cos\beta s + \frac{E_R}{R_0} \, j\sin\beta s \end{aligned}\right\} \quad (1)$$

(i) Case 1 open circuit: sub in (1)

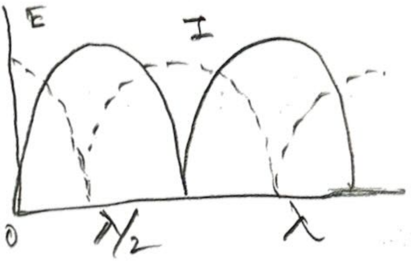


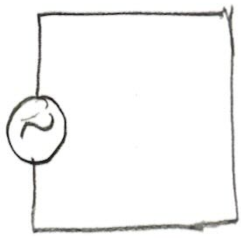




$$E_{oc} = E_R \cos\beta s$$

$$I_{oc} = j \frac{E_R}{R_0} \sin\beta s$$

Case 2: Short circuit.

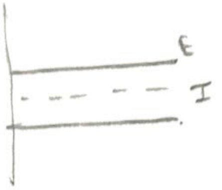



$$E_{sc} = j \, I_R R_0 \sin\beta s$$

$$I_{sc} = I_R \cos\beta s$$

Case 3: $Z_R = Z_0$





$\gamma = j\beta$

Sub $k = 0$, $Z_0 = Z_R = R_0$ in equation (1)

$$E = \frac{E_R}{2R_0}(R_0 + R_0)e^{j\beta s}$$

$$= \frac{E_R}{2R_0} \times 2R_0 e^{j\beta s}$$

$$= E_R e^{j\beta s}$$

$$I = \frac{I_R}{2R_0}(R_0 + R_0)(e^{j\beta s} - 0)$$

$$= \frac{I_R}{2R_0} \times 2R_0 \times e^{j\beta s}$$

$$I = I_R e^{j\beta s}$$

∴ There is no reflected wave then it is called as Smooth line.

5 Standing wave ($Z_R \neq Z_0$)

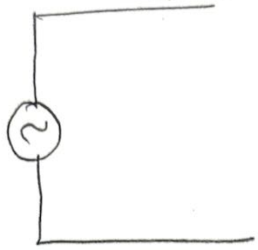
When total voltage appears to be stand till on line oscillating in magnitude with time but has fixed maxima & minimum such wave is called as Standing wave.

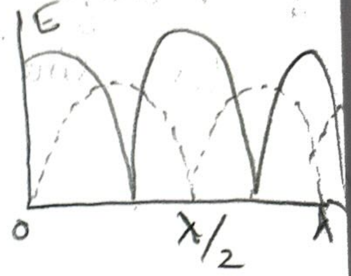
Standing wave is the combination of incidence wave & reflected wave.

$$E = E_R \cos\beta s + I_R R_0 \, j\sin\beta s$$
$$I = I_R \cos\beta s + \frac{E_R}{R_0} j\sin\beta s \qquad (1)$$

(i) Case 1 open circuit: sub in ①

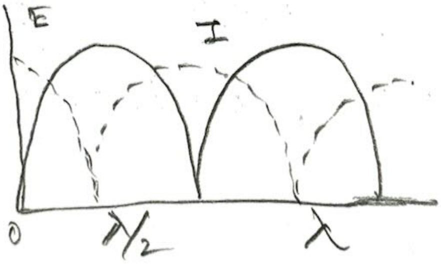
$I_R = 0$



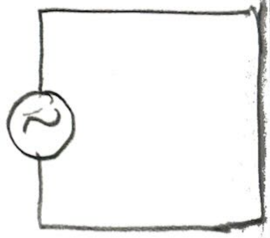



$$E_{oc} = E_R \cos\beta s$$
$$I_{oc} = j\frac{E_R}{R_0} \sin\beta s$$

Case 2: ~~S~~ Short circuit.

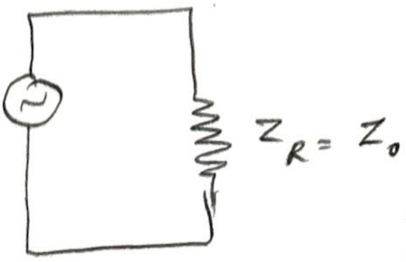



$E_R = 0$



$$E_{sc} = j I_R R_0 \sin\beta s$$
$$I_{sc} = I_R \cos\beta s$$

Case 3: $Z_R = Z_0$


$\gamma = j\beta$

Sub $k = 0$, $Z_0 = Z_R = R_0$ in equation (1)

$$E = \frac{E_R}{2R_0}(R_0 + R_0)e^{j\beta s}$$

$$= \frac{E_R}{2R_0} \times 2R_0 e^{j\beta s}$$

$$= E_R e^{j\beta s}$$

$$I = \frac{I_R}{2R_0}(R_0 + R_0)(e^{j\beta s} - 0)$$

$$= \frac{I_R}{2R_0} \times 2R_0 \times e^{j\beta s}$$

$$I = I_R e^{j\beta s}$$

∴ There is no reflected wave then it is called as Smooth line.

5 Standing wave $(Z_R \neq Z_0)$

When total voltage appears to be stand till on line oscillating in magnitude with time but has fixed maxima & minimum such wave is called as Standing wave.

Standing wave is the combination of incidence wave & reflected wave.

## Standing wave Ratio (SWR)

The ratio of the maximum to minimum magnitude of voltage ~~and~~ or current on a line having standing wave then it is called as standing wave ratio.

$$S = \left| \frac{E_{max}}{E_{min}} \right| = \left| \frac{I_{max}}{I_{min}} \right|$$

If SWR is measured across RF volt metre across the line its ratio of maximum magnitude of voltage to the minimum magnitude of voltage is called VSWR,

If SWR is measured using RF ammetre in series with line, its ratio of maximum magnitude of current to the minimum magnitude of current is called CSWR

7. Node & Antinode:

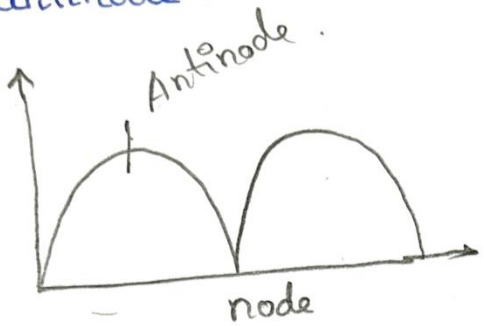
Node:

The point along line where magnitude

of V, I is zero is called Node.

Antinode:

The point along line where magnitude of V, I is maximum is called antinode.




8 Relationship between SWR and Reflection efficient.

$$E = E_R \cosh j\beta s + I_R Z_0 \sinh j\beta s$$

$$E = E_R \left(Z_R + Z_0\right)\left(e^{j\beta s} + k\, e^{-j\beta s}\right)$$

At max volt, (in phase)

$$E_{max} = \frac{E_R}{2Z_R}\left(Z_R + Z_0\right)\left[1 + |k|\right]$$

At min volt, (out of phase)

$$E_{min} = \frac{E_R}{2Z_R}\left(Z_R + Z_0\right)\left(1 - |k|\right)$$

$$S = \frac{E_{max}}{E_{min}}$$

$$= \frac{E_R/2Z_R \left(Z_R + Z_0\right)\left(1 + |k|\right)}{E_R/2Z_R \left(Z_R + Z_0\right)\left(1 - |k|\right)}$$

$|K| \Rightarrow$ Reflection co-efficient.

$$\frac{S-1}{S+1} = \frac{\frac{1+|K|}{1-|K|} - 1}{\frac{1+|K|}{1-|K|} + 1}$$

$S = \frac{1+|K|}{1-|K|}$

$$= \frac{\frac{1+|K|-1+|K|}{1-|K|}}{\frac{1+|K|+1-|K|}{1-|K|}}$$

$$= \frac{2|K|}{2} = |K|$$

$$|K| = \frac{S-1}{S+1} \Rightarrow K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$= \frac{1+|K|}{1-|K|}$$

(i) Case 1: $Z_R > Z_0$

$\frac{Z_R - Z_0}{Z_R + Z_0}$

$$S = \frac{1+|K|}{1-|K|} = \frac{1 + \frac{Z_R - Z_0}{Z_R + Z_0}}{1 - \frac{Z_R - Z_0}{Z_R + Z_0}}$$

$$= \frac{\frac{Z_R + Z_0 + Z_R - Z_0}{Z_R + Z_0}}{\frac{Z_R + Z_0 - Z_R + Z_0}{Z_R + Z_0}}$$

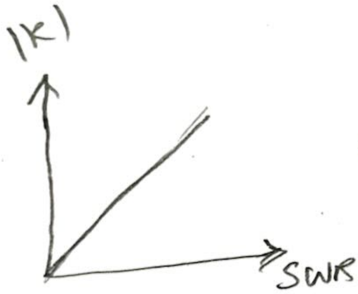
$$= \frac{2Z_R}{2Z_0}$$

$$= \frac{Z_R}{Z_0}$$

ii) case 2 : $Z_R < Z_0$

$$S = \frac{1-|K|}{1+|K|} = \frac{1 - \frac{Z_R - Z_0}{Z_R + Z_0}}{1 + \frac{Z_R - Z_0}{Z_R + Z_0}}$$

$$= \frac{\frac{Z_R + Z_0 - Z_R + Z_0}{Z_R + Z_0}}{\frac{Z_R + Z_0 + Z_R - Z_0}{Z_R + Z_0}}$$




$$= \frac{2Z_0}{2Z_R}$$

$$= \frac{Z_0}{Z_R}.$$

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9 Input impedance of dissipation less line :

$$E_S = E_R \cos\beta s + j I_R R_0 \sin\beta s$$

$$I_S = I_R \cos\beta s + j \frac{E_R}{R_0} \sin\beta s$$

$$Z_{in} = \frac{E_S}{I_S} = \frac{E_R \cos\beta s + j I_R R_0 \sin\beta s}{\left[I_R R_0 \cos\beta s + j E_R \sin\beta s\right] / R_0}$$

$$= \frac{R_0 \left[E_R \cos\beta s + j I_R R_0 \sin\beta s\right]}{I_R R_0 \cos\beta s + j E_R \sin\beta s}$$

$$= \frac{R_0 \cos\beta s \left[E_R + j I_R R_0 \frac{\sin\beta s}{\cos\beta s}\right]}{\cos\beta s \left[I_R R_0 + j E_R \frac{\sin\beta s}{\cos\beta s}\right]}$$

$$= \frac{R_0 \left[E_R + j I_R R_0 \tan\beta s\right]}{\left[I_R R_0 + j E_R \tan\beta s\right]}$$

$$= \frac{R_0 I_R \left[\frac{E_R}{I_R} + j R_0 \tan\beta s\right]}{I_R \left[R_0 + j \frac{E_R}{I_R} \tan\beta s\right]}$$

$$Z_{in} = \frac{R_0 \left[Z_R + j R_0 \tan\beta s\right]}{R_0 + j Z_R \tan\beta s}$$

i) For Short Circuit. $Z_R = 0$

$$Z_{SC} = \frac{R_0 \left[j R_0 \tan\beta s\right]}{R_0}$$

$$= j R_0 \tan\beta s.$$

$\beta = 2\pi/\lambda$

$Z_{SC}$ is purely imaginary it ~~will~~ can be denoted by $X_s$

$R + jX$

$$Z_{SC} = jX_s = jR_0 \tan\left(\frac{2\pi}{\lambda}\right)s$$

$$\frac{X_s}{R_0} = \tan\left(\frac{2\pi}{\lambda}\right)s$$

$0 = 0$

$\tan\frac{\pi}{2} = \infty$

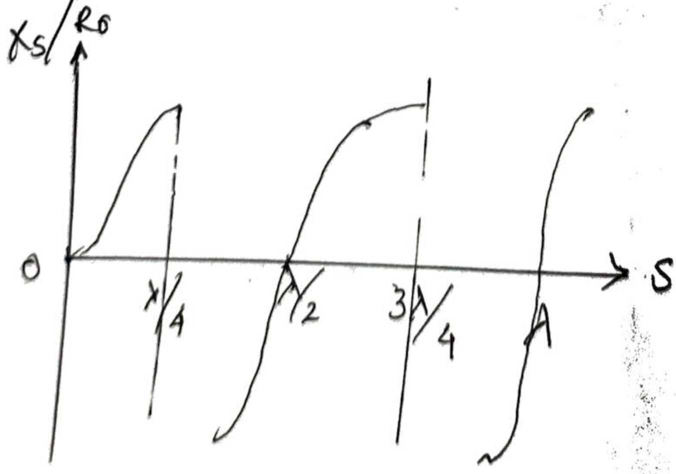
(i) $s = 0$ ; $\frac{X_s}{R_0}\ \tan(0) = 0$

ii) $s = \lambda/4$ , $\frac{X_s}{R_0} = \tan\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{4}\right) = \infty$

iii) $s = \lambda/2$ , $\frac{X_s}{R_0} = \tan\left(\frac{2\pi}{\lambda} \times \lambda/2\right) = 0$

iv) $s = 3\lambda/4$ , $\frac{X_s}{R_0} = \tan\left(\frac{2\pi}{\lambda} \times \frac{3\lambda}{4}\right) = \infty$

v) $s = \lambda$ , $\frac{X_s}{R_0} = \tan\left(\frac{2\pi}{\lambda} \times \lambda\right) = 0$



ii) For open circuit $Z_R = \infty$

$$Z_S = \frac{R_0 [Z_R + j R_0 \tan\beta s]}{R_0 + j Z_R \tan\beta s}$$

$$Z_S = \frac{Z_0 Z_R \left[1 + j \frac{R_0}{Z_R} \tan\beta s\right]}{Z_R \left[\frac{R_0}{Z_R} + j \tan\beta s\right]}$$

$$Z_S = \frac{R_0 \left[1 + j \frac{R_0}{Z_R} \tan\beta s\right]}{\frac{R_0}{Z_R} + j \tan\beta s}$$

$$Z_{OC} = \frac{R_0}{j \tan\beta s}$$

$$Z_{OC} = -j R_0 \cot\beta s$$

$Z_{OC}$ is purely imaginary it can be denoted as $X_{OC}$

$$Z_{OC} = j X_{OC} = -j R_0 \cot\beta s$$

$$\frac{X_{OC}}{R_0} = -\cot\beta s = -\cot \frac{2\pi}{\lambda} s$$

$\cot 0 = \infty$
$\cot\left(\frac{\pi}{2}\right) = 0$

(i) $S = 0$, $\frac{X_{oc}}{R_0} = -\cot\left(\frac{2\pi}{\lambda} \times 0\right)$

$= +\infty$

ii) $S = \lambda/4$, $\frac{X_{oc}}{R_0} = -\cot\left(\frac{2\pi}{\lambda} \times \lambda/4\right)$

$= 0$

iii) $S = \lambda/2$, $\frac{X_{oc}}{R_0} = -\cot\left(\frac{2\pi}{\lambda} \times \lambda/2\right)$

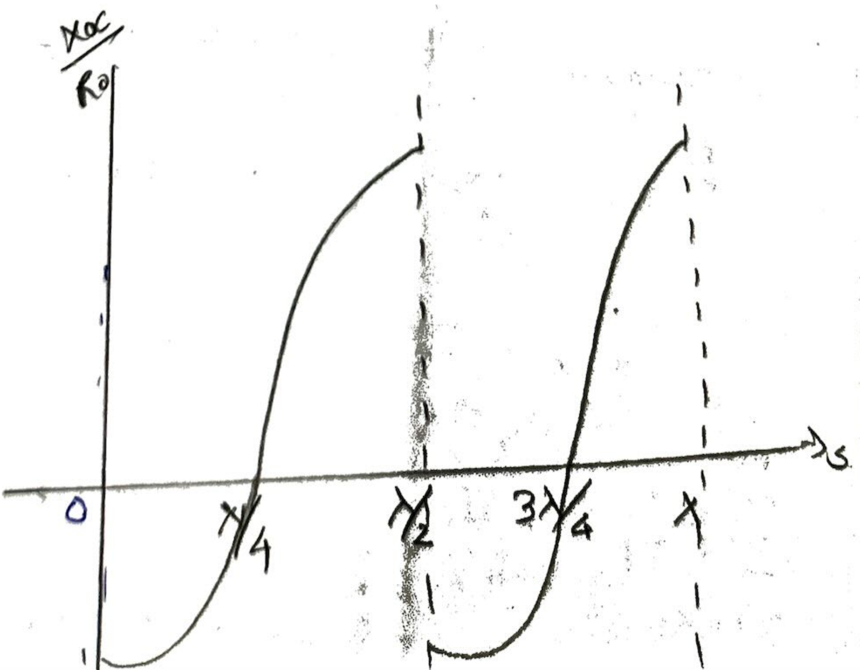
$= -\infty$

iv) $S = 3\lambda/4$, $\frac{X_{oc}}{R_0} = -\cot\left(\frac{2\pi}{\lambda} \times \frac{3\lambda}{4}\right)$

$= 0$

v) $S = \lambda$, $\frac{X_{oc}}{R_0} = -\cot\left(\frac{2\pi}{\lambda} \times \lambda\right)$

$= -\infty$



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10. Power and impedance measurement in line,

The voltage and current dissipation.

$$E = \frac{E_R (Z_R + Z_0)}{2 Z_R} \left[ e^{j\beta s} + k e^{-j\beta s} \right]$$

$$I = \frac{I_R}{2 Z_0} (Z_R + Z_0) \left( e^{j\beta s} + k e^{-j\beta s} \right)$$

$$Z_R = \frac{E_R}{I_R} \; ; \; Z_0 = R_0$$

$$E = \frac{E_R}{2 E_R / I_R} (Z_R + Z_0) \left( e^{j\beta s} + k e^{-j\beta s} \right)$$

$$= \frac{I_R}{2} (Z_R + Z_0) \left( e^{j\beta s} + k e^{-j\beta s} \right)$$

$$I = \frac{I_R}{2 R_0} (Z_R + Z_0) \left( e^{j\beta s} + k e^{-j\beta s} \right)$$

$\Rightarrow$ maximum volt and current,

$$E_{max} = \frac{I_R}{2} (Z_R + R_0)(1 + |k|)$$

$$I_{max} = \frac{I_R}{2 R_0} (Z_R + R_0)(1 + |k|)$$

$$\frac{E_{max}}{I_{max}} = \frac{I_R/2 \, (Z_R + R_0)(1 + |k|)}{I_R/2R_0 \, (Z_R + R_0)(1 + |k|)}$$

$= R_0 = R_{max}$.

Maximum voltage & minimum current.

$$I_{min} = \frac{I_R}{2R_0} (Z_R + R_0)(1 - |k|)$$

$$\frac{E_{max}}{I_{min}} = \frac{\frac{I_R}{2} (Z_R + R_0)(1 + |k|)}{\frac{I_R}{2R_0} (Z_R + R_0)(1 - |k|)}$$

$$R_{max} = R_0 S$$

→ Minimum voltage & maximum current.

$$E_{min} = \frac{I_R}{2} (Z_R + R_0)(1 - |k|)$$

$$\frac{E_{min}}{I_{max}} = \frac{I_R/2 \, (Z_R + R_0)(1 - |k|)}{\frac{I_R}{2R_0} (Z_R + R_0)(1 + |k|)}$$

$$R_{min} = R_0 \left(\frac{1}{S}\right)$$

Minimum V & minimum I

$$\frac{E_{min}}{I_{min}} = \frac{I_R/2 \, (Z_R + R_0)(1 - |k|)}{\frac{I_R}{2R_0} (Z_R + R_0)(1 - |k|)}$$

$$= R_0$$

Power flowing into resistance.

$$P = \frac{E_{max}^2}{R_{max}} \quad , \quad P = \frac{E_{min}^2}{R_{min}}$$

$$P^2 = \frac{E_{max}^2 \, E_{min}^2}{R_0 s \; R_0/s} = \frac{E_{max}^2 \cdot E_{min}^2}{R_0^2}$$

$$P = \frac{E_{max} \, E_{min}}{R_0}$$

$$P = I_{max} \, I_{min} \, R_0$$

11. Reflection losses on unmatched line

* When characteristic impedance is not equal to load impedance, $Z_R \neq Z_0$, power will not be delived in load. But the part will be reflected.

The combination of reflected wave and incident wave leads to standing wave ratio (SWR).

$$V_{max} = V_i + V_r = \frac{I_R}{2}(Z_R + Z_0)(1 + |K|) \text{ in phase}$$

$$V_{min} = V_i - V_r = \frac{I_R}{2}(Z_R + Z_0)(1 - |K|), \text{ out of phase}$$

$$S = \left|\frac{V_{max}}{V_{min}}\right| = \left|\frac{V_i + V_r}{V_i - V_r}\right|$$

$$P = \frac{V_{max} \cdot V_{min}}{R_0}$$

$$P = \frac{(V_i + V_r)(V_i - V_r)}{R_0}$$

$$= \frac{V_i^2 - V_r^2}{R_0}$$

$P_i \Rightarrow$ incident power

$P_r \Rightarrow$ reflected power.

$$P = P_i - P_r$$

$$\frac{P}{P_i} \left(\frac{\text{power delivered to load}}{\text{incident wave power}}\right) = \frac{P_i - P_r}{P_i}$$

$$= 1 - \frac{P_r}{P_i}$$

$$1 - \frac{V_r^2 / R}{V_i^2 / R} = 1 - \frac{V_r^2}{V_i^2} = 1 - |K|^2$$